Is Synechism Necessary?

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In the eighth Cambridge Conferences Lecture, Peirce says that he “[likes] to call [his] theory Synechism, because it rests on the theory of continuity” (Peirce 1898b, p. 261); and much of his later system-building is indeed animated by the conviction that a proper understanding, and an effective deployment, of the concept of continuity are essential to a philosophy whose “principal utility . . . is to furnish a . . . conception of the universe, as a basis for the special sciences” (Peirce 1903, pp. 146–147). If continuity is as vital to Peirce’s later philosophy as he thinks it is, then that philosophy cannot be said to be fully successful by its own lights if Peirce’s conception of continuity fails to hold water. So anyone who believes, as I do, that Peirce has much to say that philosophers today need to hear, should be worried about whether his conception of continuity does hold water.

It is surprising, and disheartening, that those of us who take Peirce seriously have not been more worried about this. After all, an early classic in our field ends by asserting that Peirce’s manuscripts are not even, as they might appear to be, “the ruins of a once great structure.”

The reason is that Peirce was never able to find a way to utilize the continuum concept effectively. The magnificent synthesis which the theory of continuity seemed to promise somehow always eluded him, and the shining vision of the great system always remained a castle in the air. (Murphey 1961, p. 407)

Why has Murphey’s pessimism not cast a longer and darker shadow over subsequent work on Peirce? Perhaps now that Putnam’s paper on Peirce’s continuum (Putnam 1992) has joined The Development of Peirce’s Philosophy on the mandatory reading list, we have come to think that there is nothing amiss here that a little nonstandard analysis can’t put right.

But how can we sustain such a sanguine view, in the face of the relentless, and ultimately annihilatory, criticisms that Peirce himself leveled at
the conception that Putnam so dazzlingly reconstructs? We might as well pause for a moment here to recall what that conception is. I will follow my earlier usage (Moore 2007, pp. 425–426, 438–448) and simply call this the “Peircean conception,” which has the merit of convenience and also of acknowledging the longevity and relative thoroughness of development that sets this conception apart from the other analyses of continuity Peirce would have signed his name to at other times in his life. He summarizes this conception in the third Cambridge Conferences Lecture by defining a continuum as “a collection of so vast a multitude that in the whole universe of possibility there is not room for them to retain their distinct identities, but they become welded into one another” (Peirce 1898a, p. 174). In the pages that precede this definition Peirce begins his construction of such a collection by constructing a sequence of collections in which there is, for each infinite multitude $\kappa$, a collection of multitude $\kappa$; the union of the collections in this sequence is then shown to be a Peircean continuum. This collection is, as Peirce’s definition demands, confused: its elements do not “retain their distinct identities, but they become welded into one another.” It is also transfinite (my substitute for Peirce’s ungainly “supermultitudinous”): for any $\kappa$, a collection of $\kappa$ discontinuities could be distinguished within it. And it harmonizes with Peirce’s much earlier dictum, which is fundamental to all his subsequent thinking about continuity, that “a continuum is precisely that, every part of which has parts, in the same sense” (Peirce 1868, p. 68). Fernando Zalamea calls this the reflexivity of the Peircean continuum “since in a full continuum satisfying this reflection principle, the whole is reflected in any of its parts” (Zalamea 2010, p. 211).

To make this conception go, Peirce needed a quasi-Cantorian theory of collections, and the search for that theory occupied much of his mathematical energies for about ten years, with their midpoint around the turn of the century. As of 1903 that search was still in full swing, but just a couple of years later we find Peirce voicing the suspicion that the concept of collection is “indecomposable” (Peirce 1905, p. 209). That despairing aside occurs, as it happens, in a fragmentary exposition of his theory of continuity; and that fragment foreshadows a long addendum on continuity, written in May of 1908, to an installment in his “Amazing Mazes” series in the Monist (Peirce 1908). There Peirce abandons his collection theoretic account after more than a decade of trying to make it work, and announces that he has “taken a considerable stride toward the solution of the question of continuity, having at length clearly and minutely analyzed my own conception of a perfect continuum as well as that of an imperfect continuum” (Peirce 1908, p. 215). What comes out of that long, clear and minute analy-
sis is a more thoroughly geometrical definition of continuity, which Jérôme Havenel has aptly dubbed “topological” (Havenel 2008, pp. 117–125). It is hard to deny that there is a shift here in Peirce’s thinking, but it is equally hard to see anything well developed enough to call a new theory of continuity. The manuscript evidence we have suggests that the long, clear and minute analysis took place over a period of roughly seventy-two hours, and that Peirce never succeeded in writing out the full dress definition that he was supposedly drawing on in his published addendum. Since there is, so far as I know, no more detailed development of this theory to be found anywhere in the Peircean corpus, I do not see how to avoid the pessimistic conclusion that this final attempt at a theory of “true continuity” was a failure. Which brings us back to where we began, face to face with Murphey’s challenge to the later Peircean system as a whole.

Of course one man’s failure to lock onto a concept, even when the man in question is C.S. Peirce, does not prove that there is no concept there to be locked onto. Mathematicians of Peirce’s caliber often have intuitions that outrun the technical apparatus at their disposal. Recent work by Philip Ehrlich (Ehrlich 2010) and Fernando Zalamea (Zalamea 2003) would seem to constitute at least a partial vindication of Peirce’s theories of continuity. A total vindication, grounded in mathematical developments Peirce can hardly have foreseen, is naturally out of the question. But we need not insist on a rigorous reconstruction of exactly Peirce’s conception (supposing for the moment that exactness is even something we can aspire to in this case). A rigorous theory of continuity that does all, or even most, of the philosophical work Peirce wanted his theory to do, would surely be total enough, as vindications go. But with all due respect for the good technical work done by Ehrlich, Zalamea and the rest, I do not see that we are there yet.

We may one day get there. I confess that to me it looks like an impossibly tall order, but perhaps that is just a failure of imagination on my part. Be that as it may, there are compelling reasons to see what drops out, and what remains, of Peirce’s later philosophy if we have to make do without some reasonable facsimile of his continuum. We have just seen one reason to look into this: as long as so much of that philosophy hinges on a promissory note, its appeal as a source of insight into living philosophical problems will be limited at best. Here is another. We who are already convinced of Peirce’s greatness tend to forget how impenetrable he can look to the rest of the human race. The conceptual overhead of Peirce’s philosophy is almost prohibitively high, even without the continuum. By my own reckoning we will need at least something answering to his categories and his
theory of signs if we are to get much philosophical mileage out of Peirce. (Your mileage, of course, may vary.) That is already quite a lot to digest. Why pile the continuum on top of that if we don’t have to?

I have been talking blithely of Peirce’s “system” and his “later philosophy” as if we all knew exactly what these phrases mean. Without trying to be as precise as one can hope to be about that, I will simply renounce any ambition to make grand claims about continuity in Peirce’s later thought or system, and content myself with gauging the need for his continuum in what surely counts, for all its undoubted flaws, as a masterpiece of his final decades, the Harvard Lectures on pragmatism from 1903. In fact, because time is so short, I will only be able to examine one brief and very important passage from Lecture IV.

That passage opens the second of two numbered sections whose primary purpose is to argue “that Thirdness is operative in Nature” (H4, 181). Section 1 (181–183) contains the argument itself; in Section 2 Peirce explains “how [he connects] generality with Thirdness” (183). The explanation begins with the traditional Aristotelian definition of the general as that which is sayable of many things (de multis). Thus “the general is essentially predicative and therefore of the nature of a representamen.” Attentive members of the audience would have remembered that Peirce had introduced Thirdness in Lecture III as “the Idea of that which is such as it is as being a Third, or Medium, between a Second and its First. That is to say, it is Representation as an element of the Phenomenon” (H3, 160). This would have given them some idea, but only a very general one, of how he was about to connect generality with Thirdness. He proceeds, in the two very densely argued paragraphs that follow, in three phases. A few general comments on the forest may help us keep our bearings, once we have plunged into the thick of all those trees. I contend, first of all, that the arguments in all three phases depend for much of their supposed force on the availability of a fully worked out Peircean conception of continuity. But, second, even if such a conception were available, the arguments would turn out in the final analysis to be not especially compelling abductions, which advance hypotheses that meet no genuinely pressing explanatory need.

The first phase of Peirce’s explanation points out an alleged defect in the traditional definition of generality, and proposes transinfinity as a remedy:

In another respect, however, the definition represents a very degenerate form of generality. None of the scholastic logics fails to explain that sol is a general term, because although there happens to be but one sun yet the term sol aptum natum est dici de
multis. But that is most inadequately expressed. [P1] If sol is apt to be predicated of many, it is apt to be predicated of any multitude however great, and since [P2] there is no maximum multitude, [C] those objects of which it is fit to be predicated form an aggregate that exceeds all multitude.

(The bracketed premise and conclusion labels are mine.) The conclusion (C), which mentions transinfinite aggregates, clearly presupposes that there are Peircean continua, and that presupposition turns out to be just as essential to the logic of the argument. Sticking just to what Peirce explicitly says, the argument is so egregiously bad that a logician of Peirce’s caliber cannot have thought for a moment that it was successful. Even if, for any (finite or infinite) multitude \( \kappa \), there could be \( \kappa \) cats, it does not follow that there is a transinfinite aggregate of cats.

It doesn’t look quite so bad if we make the modalities explicit and help ourselves to arguments found elsewhere in the Peircean corpus. The need for some modal operators is clear enough from the Aristotelian formula itself, which says that a general term is aptum to be predicated of many, not that it is predicated of many; and Peirce’s remarks on multitude further reinforce the importance of possible instances. Once a few well-placed modalities have put the actual collection of cats on the sidelines, we are free to carry out the construction, summarized above, that Peirce uses to build his continuum from a sequence of possible collections in which every infinite multitude is represented. Peirce’s comments on the Aristotelian formula, and on the generality of ‘sol’, justify the shift to possible collections of instances. If we don’t look at this too hard, we can see how Peirce might have thought he had the raw materials for the construction of a transinfinite aggregate of instances. If there is another way to make the argument of this first phase even halfway respectable, someone else will have to find it; this is the best that I can do.

Unfortunately the construction is no good, so the argument is no more than halfway respectable. But even if it were a smashing success, and even if we were convinced that the possible instances of any universal formed a continuous aggregate, we could still refuse to identify the universal with that aggregate. Peirce never, so far as I know, gives any indication that he sees this gap, or makes any attempt to bridge it. I can only conclude that he made an abductive leap here, and that this was one of those cases he takes note of in Lecture VII (228–231), in which abductions are mistaken for perceptions. The hypothesis that a universal is a continuum of potential instances certainly explains why a universal has a continuous aggregate
of potential instances. And one can see why a philosopher whose mathe-
matizing and system-building impulses were so overpowering could work
that hypothesis for all it was worth, as Peirce unquestionably did. But from
a point of view less strongly colored by those impulses, the hypothesis
looks much less compelling. Surely it is just as natural, if not more so, hav-
ing refused to reduce universals to collections of their instances, to deny
that a universal is any kind of aggregate at all. What residual explananda
does that denial leave us with, that would drive us into the arms of the
Peircean alternative?

The second phase of Peirce’s argument paves the way for representa-
tion as the third person of the one being that is also continuity and gener-
ality. The new and crucial idea is an ordering property of the transinsfinite
aggregate of instances that constitutes a general:

Take any two possible objects that might be called *suns* and
however much alike they may be, any multitude whatsoever
of intermediate suns are alternatively possible and therefore,
as before, these intermediate possible suns transcend all multi-
tude. In short, the idea of a general involves the idea of possible
variations which no multitude of existent things could exhaust
but would leave between any two not merely *many* possibilities,
but possibilities absolutely beyond all multitude. (H4, 183)

Peirce’s thought here is fairly easy to follow if we stick to very simple ex-
amples, though it is far from trivial to apply it in more complicated cases.
Consider two ping-pong balls off the same assembly line. Their qualita-
tive near-identity can be rendered total by continuous transformation of
the shape, size, etc., of Ball A so that each of these qualities exactly matches
the corresponding quality of Ball B.

Peirce’s point here is one worth making, but it can be made without
bringing in his continuum. We can make some kind of rough and ready
sense of the idea of possible instances between two instances of a general;
and if we are sufficiently unapologetic in our realism, we can recognize the
arbitrariness of any upper limit one might put on the number of instances
there could be between any two (even if we also know that there is some
limit, set by the laws of nature that actually obtain). That does not quite
convince me that there could be an arbitrarily large collection of instances
in between any two: we may be unable to identify an upper bound, not be-
cause there is none, but because our epistemic limitations prevent us from
figuring out what the limit is. But even if there is no such bound, we have
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no more reason here than we did in the first phase of the argument to conclude that the aggregate of intermediate instances is a Peircean continuum. The analysis founders on the same rocks: on the inference from the several collections of all infinite multitudes to the one transinfinite aggregate, and on the identification of the aggregate of possible instances of a general with the general itself.

In phase three Peirce closes the circle by coming back around to representation, where the first phase began:

Now Thirdness is nothing but the character of an object which embodies Betweenness or Mediation in its simplest and most rudimentary form; and I use it as the name of that element of the phenomenon which is predominant wherever Mediation is predominant, and which reaches its fullness in Representation.

The betweenness that emerged in phase two as a defining feature of generals, rightly (that is to say, synechistically) understood, emerges in phase three, under the name of Mediation, as the common defining feature of generality and representation. For this to fly as serious metaphysics, we need more than broad analogies, and once again it is Peirce’s continuum that is supposed to hold everything together. Without his continuum, the analogies would have little to recommend them, and yet they would be all we had.

There appear, furthermore, to be serious disanalogies between the mediation performed by points in a continuous line (call this geometrical betweenness) and that performed by potential instances of a universal (call this generic betweenness). The standard axioms for geometrical betweenness imply that if the points \( C \) and \( D \) are between the points \( A \) and \( B \), then \( C \) is either between \( A \) and \( D \) or between \( B \) and \( D \). This does not hold for generic betweenness. For example, Barack Obama and I are both instances of the universal human being but I am (alas) shorter and heavier than Obama is. There is clearly a possible \( M_w \) identical to me in every respect except for weighing a bit less, so that his weight would be in between Obama’s and mine. Similarly there is a possible \( M_h \) who differs from me only in having a height somewhere between Obama’s and mine. \( M_w \) and \( M_h \) are both generically between Obama and me. But \( M_w \) is lighter than both \( M_h \) and me, and hence cannot be generically between us; and he is shorter than both \( M_h \) and Obama, so he cannot be generically between them. This argument is not iron-clad: one could question, for example, the definition of generic betweenness upon which it implicitly rests—that \( Q \) is generically between \( P \) and \( R \) iff it is qualitatively identical to \( P \) except for being more similar to
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$R$ in some respect than $P$ is and also more similar to $P$ in that respect than $R$ is. This is surely disputable as a precise explication of what Peirce had in mind when he talked of intermediate instances. The problem—which I am happy to bequeath to the reader, along with the burden of proof—is to define generic betweenness in a way that is faithful to Peirce’s conception, and at the same time makes it as closely analogous to geometrical betweenness as Peirce needs it to be.

Must we, then, just abandon all hope? If in Section 2 and other key passages from the Harvard Lectures Peirce really does need, as I have been urging, a continuum concept that he never succeeded in working out, does that mean that the philosophical system laid out in the lectures is no more than a castle in the air? By no means! Remember, first of all, that this handful of lines from Section 2 makes up a very small fraction of the lectures as a whole. And it is a gloss on the main argument in Section 1 for the reality of Thirdness, an argument that works just as well without the gloss.

Peirce begins Section 1 by telling his audience that he will “argue that Thirdness is operative in Nature,” and proposes that they “attack the question experimentally” (181). He goes on to announce what he readily concedes to be “a very silly experiment,” that of dropping a small stone, which falls to the floor, as everyone in the room already knew that it would. That experiment hardly proves that Thirdness is operative in nature. But that experiment is subsidiary to another, which does support Peirce’s contention, and to which the actual dropping of the stone is tacked on as a superfluous afterthought. This latter experiment consists in announcing that he will drop the stone, asserting that he can correctly predict the result, offering to prove that by actually dropping the stone, and then observing the effect on his audience:

Here is a stone. Now I place that stone where there will be no obstacle between it and the floor, and I will predict with confidence that as soon as I let go my hold upon the stone it will fall to the floor. I will prove that I can make a correct prediction by actual trial if you like. But I see by all your faces that you all think it will be a very silly experiment. Why so? Because you all know very well that I can predict what will happen, and that the fact will verify my prediction.

It is the last sentence that states the hypothesis that Peirce’s experiment confirms: that everyone in his audience knows that the stone will fall. He confirms it further—and in a manner especially well suited to a lecture on pragmatism—by offering to lay a wager on the outcome of releasing the
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stone, at a hundred to one, and noting that no one takes him up on the offer.

These experimental results are the basis of Peirce’s argument “that Thirdness is operative in nature.” He distinguishes between regularities that arise by chance and those in which “some principle or cause is really operative” (182). His audience’s reactions to Peirce’s experimental stimuli show that they believe in particular that “the regularity with which stones have fallen has been due to some active general principle,” whence it follows that they accept the thesis that Peirce set out to prove: that Thirdness is operative in nature. Indeed, “every sane man will adopt . . . the hypothesis” that an active general principle is operative when the stone falls, and is operative in futuro even when the stone is still suspended, because (as every sane person knows), the stone would fall if it were released.

There is a wealth of philosophical substance in these few pages of Lecture IV, far more than I can begin to do justice to. My present interest in the passage is entirely methodological. Peirce rests his case for the reality of Thirdness, not on his continuum, but on the proto-scientific common sense of his audience. I say “proto-scientific,” because the distinction between accidental and lawlike generalizations is obviously of great scientific importance, and also surely more ancient (under whatever name, if any) than the refinement of common sense that we know as science, properly so called. Which is to say, in the philosophical jargon that we favor nowadays, that the argument of Section 1 is a naturalistic one.

That is of course not to say that Peirce is doing exactly what Quine, whose understanding of naturalism is now the default, would do with the same material. Quine is the apothecosis of that nominalistic strain of naturalism that runs through Human and Mill; he is the shining example of the virtues and limitations of this great tradition. Peirce knew this tradition well, and gave it all the large measure of respect that it is due. But his view of it was informed by an intimate acquaintance with Kant, with medieval logic, and with the living realities of scientific practice. Peirce and Quine are both anti-Cartesians: where the former refuses to take “paper doubts” seriously, the latter stresses the impossibility of repairing Neurath’s boat anywhere but on the open sea. In particular both deprecate “the intellectual dishonesty of denying the existence of what one daily presupposes” (Putnam 1979, p. 347). Peirce’s case for Thirdness pretty much boils down to accusing anyone who acknowledges that he can predict that the stone will fall, but pretends to doubt the reality of Thirdness, of just such dishonesty. If these shared doctrines add up to naturalism, then Peirce and Quine both deserve to be called naturalists.
They quickly part the ways, of course, when it comes time to act on these methodological generalities. What matters most to Peirce are precisely the points at which, from a Quinean standpoint, he goes most radically wrong: Peirce’s insistence on the reality of Firstness and Thirdness puts him fundamentally at odds with Quine’s predominant urge to flee from intension. For his part, Peirce criticizes that urge, as personified by Mill (the leading nominalist of his age, as Quine is of ours) on naturalistic grounds: the trouble with nominalism is that it blinds its adherents to those aspects of the science we actually possess that do not fall in with their philosophical preconceptions. He levels that charge at Mill in his recollection, at the end of Lecture II, of a conversation he had with Chauncey Wright soon after the publication of *The Origin of Species*. Peirce told Wright that Darwin’s ideas of development had more vitality by far than any of [Wright’s] other favorite conceptions and that though they might at the moment be in his mind like a little vine clinging to the tree of Associationalism, yet after a time that vine would inevitably kill the tree … [because] Mill’s doctrine was nothing but a metaphysical point of view to which Darwin’s, which was nourished by positive observation, must be deadly. (Peirce 1903, p. 158)

He then generalizes the point by opposing the realistic instincts of working scientists to the nominalistic prejudices of “mere reporters”:

All nature abounds in proofs of other influences than merely mechanical action even in the physical world. … [As for] men whose lives are mostly passed within the four walls of a physical laboratory … the more clearly they understand how physical forces work the more incredible it seems to them that such action should explain what happens out of doors. A larger proportion of materialists and agnostics is to be found among the thinking physiologists and other naturalists, and the largest proportion of all among those who derive their ideas of physical science from reading popular books. These last, the Spencers, the Youmans, and the like, seem to be prepossessed with the idea that science has got the universe pretty well ciphered down to a fine point; while the Faradays and the Newtons seem to themselves like children who have picked up a few pretty pebbles upon the beach.
I leave it as an exercise for the reader to identify the Spencers and the Youmans of our own day.

So now to return at last to the question with which we began: how deeply should we descend into Murpheyan pessimism about Peirce’s late philosophical system? On the one hand, if my analysis is correct, the errors in Peirce’s thinking about continuity are serious and they do serious damage. On the other, the arguments those errors vitiate can in some cases be dropped without serious loss, and in others can be replaced with arguments from Peirce himself or, failing that, with altogether new ones. I have reviewed one case in which things do work out that way. But that barely begins the long and hard labor that will have to go into a more comprehensive estimate of the damage and the prospects for repairing it.

With that large caveat in mind, my own attitude is fairly optimistic. It can perhaps be best explained by contrast with two extreme assessments of Peirce that can be found in the secondary literature (that is, in the literature that assumes that Peirce is worth studying in the first place). There is first of all the reverence for Peirce as the Pope of Milford, whose pronouncements set forth the whole truth and nothing but the truth. As Peirce himself says, that is no way to read any philosopher. It is especially ill-suited to the reading of one who was, when he reached the peak of his powers, also embittered, exhausted and in a desperate hurry to make the big splash that would win him the position and the public profile he deserved.

It is easy to be driven to the other extreme when we realize that the grand system Peirce lays before us in the Harvard Lectures does not hang together as he thought it did, because the kinks in his theory of continuity could not be worked out. Why not just ignore Peirce’s architectonic pretensions, then, and enjoy the flashes of brilliant light against the Cimmerian darkness, without trying to fit them together into some larger unity? With the heyday of Wittgensteinianism so far behind us, it is no longer the compliment it once was to say that a philosopher works piecemeal. But whether a compliment or not, to say that about Peirce is to say that he failed in his own terms: Murphey was quite right to stress from his introduction onward that Peirce was dead serious about his architectonic.

And rightly so, I say. While parts of the system are impressive enough when taken in isolation—the theory of signs, the categories, the pragmatism, the logic of perception—they are even more impressive when they work together as Peirce intended them to. I work on Peirce because I believe that his mature realism, and the accompanying critiques of nominalism, are abundant and underutilized resources for those of us who think that logical empiricism has now permanently run aground. But his real-
ism is not a discrete claim that can be stated and assessed in isolation from the system as a whole: the realism is the system, a complex web of mutually reinforcing epistemological and metaphysical commitments. We therefore cannot tap these resources without an accurate assessment—one that overstates neither its weaknesses nor its strengths—of Peirce’s system in its most finished form. I have tried to encourage the hope that any gaps in the system that open up when we give up on Peirce’s continuum are either harmless, or can be rendered so by making adjustments to the system, or can be filled with materials imported from elsewhere if not already supplied by Peirce himself.

We wind up with an attitude that should have had a lot going for it in advance of any detailed examination of the vicissitudes of the Peircean continuum; for it is the attitude we should bring to the study of any great figure from the past. Peirce has left us, not any kind of final word, but a work in progress, one eminently worth carrying on, in the spirit of the one who started it. Which is to say that we must as resolutely critical, and as ruthless in paring away what does not work, as Peirce was at his best.

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