

## A cournotian approach to the emergence of relational collectives

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### Abstract

Computational processes in cellular networks are supported by complex molecular and ionic mechanisms, composing a distributed semi-parallel architecture where each processing unit has partial information about the others. How do cognitive functions of organisms emerge from this kind of computational process? The concept of *Cournotian Process* characterizes the kind of interaction that occurs between multiple independent factors eliciting the emergence of a common product. As in the classical concept of *chance* advanced by Cournot (1838), the meeting of independent causal lines (i.e. semi-deterministic processes) generates a result that cannot be described as a *function* of the factors, but as a *relational* collective. In this work we suggest that the resulting *cognitive functions* should be described by *mathematical relations* instead of mathematical functions, and that the inference from brain structures to mental activity is a semantic reasoning based on *similarities*.

**Key Words:** *emergence, Cournotian processes, cognitive functions, relational collectives, similarity.*

## 1 - Introduction

According to Information and Systems Theories, an empirical measurement involves an interaction between an observer, in some extent also a receiver of information, and a physical system, taken as the source of information. The physical system has a temporal dynamics that can be partially captured by means of a series of measurements.

In order to explain and predict phenomena, the observer elaborates mathematical theories, aimed to cover the temporal dynamics. In the classical deterministic framework, the dynamics is assumed to be completely described by mathematical *functions* (one-to-one or many-to-one mapping of first-order relations), which could in principle be computed by a single Turing machine.

In this paper, we argue that biological and cognitive processes occurring in systems composed of partially independent units should be conceived as mathematical *relations* (including one-to-many mappings) called *Cournotian Processes*. Initially, we distinguish mathematical relations from functions, in the context of their usage in empirical science.

The classical Newtonian schema of causation inspired the functional concept of causality (Mackie, 1972):

$$\text{Effect} = \mathbf{f} (\text{Cause 1, Cause 2, Cause 3,....})$$

where  $\mathbf{f}$  specifies how the set of causes relate to the effect. Causes and Effect are conceived as physical changes that occur in the system, relative to its initial and boundary conditions.

Mathematical functions describe a univocal relation between the causes and the effect. How do the causes relate to each other? There are two possibilities:

- a) they are previously correlated, such that there is a function  $\mathbf{F}$  that deduces  $\mathbf{f}$ ;
- b) they are not previously related, and therefore a function  $\mathbf{F}$  that deduces  $\mathbf{f}$  *does not exist previously*; i.e.,  $\mathbf{f}$  comes to existence only at the moment when the causes interact. This possibility is illustrated by the concept of



"chance" as *an absence of previous correlation of causal chains*, according to the proposal advanced by A. Cournot (1838).

Biological processes in a cell (e.g. metabolic networks) or in the whole organism (e.g., different tissues and systems working autonomously) are parallel in a stronger sense than in classical computation, since they contain *simultaneous phenomena that cannot be composed or decomposed in sequential processes*. This aspect has been recognized in several approaches, as probability theory, fuzzy logic, and non-linear thermodynamics of dissipative structures.

Another typical factor present in biological computation is that the mechanisms amenable to be described by algorithms have a property of self-organization, conferring a spontaneity to their dynamic evolution. A central aspect of such processes, is *the availability, for each parallel processing unit, of information about the states of the other units*. However, because of architectural and (possibly) general physical constraints (as e.g. the finite velocity of signal transmission), information about the system available for each unit is partial. The global, self-organizing processes that characterize such systems is based on the partial information that each processing unit has about the others. In this sense, self-organization is also an internal learning process by which the parts of the system may increase their degree of communication and become more integrated.

The motivation to develop our approach comes from difficulties in understanding cellular information processing in terms of classical Connectionism, and, with more reason, in terms of a sequential model. Weng, Bhalla and Ivenger (1999) note that, even in an extremely simplified approach, the arrangements of signaling molecules are similar to redundant functions, although their kinetic properties can be very different. The authors suggest that in the absence of complete knowledge of the quantitative parameters of these systems, it is possible to reach a rough perspective of their evolution by analyzing the *linear* pathways.

A serious difficulty found in this strategy is the interaction of each linear pathway with other pathways, sometimes from distant regions of the cell. This branching of the pathways into molecular networks is not completely solved in the classical Connectionist model, where a network is analyzed into parallel connections between a large number of interconnected conventional processors, thus dividing the process in several *threads* (Hillis, 1985). The network then executes a parallel algorithm (Cormen, 2002, p. 688ff) that is more complex than the sequential ones, but still in the context of a classical Turing machine.

As biological systems are able to process information independently of an external controlling agent, but depending on the capacity of communication

between the cells, Cournot's reasoning about the interaction of independent deterministic processes to generate a new, partially unpredictable phenomenon, seems to be a plausible approach to this biological spontaneity. His intuition was that a non-deterministic result could emerge from the interaction of deterministic processes that evolve independently of each other until the moment when they interact.

This intuition is formalized here. First, we characterize a spatio-temporal process evolving in time, under the constraints of classical physics.

## 2- Formalization of Dynamical Processes

Consider a biophysical process **P**, consisting of sub-processes internal to the brain, their mutual interactions, and their interactions with the body and the environment. The system where **P** occurs is named **S** (it is, of course, larger than the brain). Also consider a cognitive process **M** that emerges from **P**.

**M** *strongly emerges* from **P** in **S** IFF:

1. There is a constructive method to apply the laws (**L**<sub>1</sub>, ..., **L**<sub>n</sub>) that rule the evolution of **S** (i.e., the laws of physics, chemistry and biology), when the initial (**S**(**0**) = **f**) and boundary conditions (**S**(**a**) = **k**, **S**(**b**) = **k'**, etc.) are known;
2. If **E** is a state of **P**, then the probability of obtaining a complete description of **E** from **L**<sub>i</sub> applied to **S**(**t**) = **f\***, keeping the boundary conditions constant (or computing all their variations and their consequences), is lower or equal to the probability of obtaining **E** from a simple random sampling.

In other words, **M** is *strongly emergent* in **S** if the laws of the system cannot *predict* a state with more accuracy than by chance. This situation is colloquially expressed in the statement: "it was not possible to *deduce* the states of **P** based on the laws of **S**". For instance, the process of memory retrieval possibly is strongly emergent relatively to brain activity, because the (complete) knowledge of the laws of neuroscience, together with (complete) knowledge of initial and boundary conditions, would not be sufficient to predict the memory trace that a subject is going to remember.

Knowing the *structural restrictions* that rule the evolution of a system may help to predict what is going to happen, with a higher probability than by chance.

This possibility has been frequently used in scientific research in cognitive neurobiology; e.g., when genetically modified mice do not express a protein with a specific role in brain processes, it is possible to predict the corresponding deficit in mental functioning (see e.g. Mayford et al., 1996). In order to clarify how a structuralist view can contribute to the understanding of mental emergence, we begin by defining the concept of *structure*.

A structure can be conceived as a model-theoretic *set* subject to some *restrictions*. More precisely, the definition of a structure requires the reference to a *family of basic sets*, possessing the elements under restriction. Such families of sets are called *universes*. The simplest structures, called *mono-sorted*, are those that possess only one universe. As the universes correspond to classes of objects, it is usual to call a structure with  $n$  universes a *many-sorted structure*. For practical purposes, in empirical sciences, we use only enumerable structures (including, in some cases, infinite structures).

Consider an enumerable structure  $\mathbf{A}$ . It is possible to define an infinite number of structures having  $\mathbf{A}$  as their universe. In order to define a specific structure  $\mathbf{E}(\mathbf{A})$ , some definitions are needed. For each  $n$ , let  $\mathbf{A}^n$  denote the  $n$ -th Cartesian power product of  $\mathbf{A}$ .  $\mathbf{P}(\mathbf{B})$  is a set composed by all parts of any power set  $\mathbf{B}$  (for  $\mathbf{B} = \mathbf{A}, \mathbf{A}^2, \dots$  etc.). Consider, for each  $h=1, \dots, n$ , the union of the sets:

$$\mathbf{P}(\mathbf{A}^1) \cup \dots \cup \mathbf{P}(\mathbf{A}^h) \cup \dots \cup \mathbf{P}(\mathbf{A}^n)$$

The union is indicated by  $\mathbf{A}^*$ .  $\mathbf{R}$  belongs to  $\mathbf{A}^*$ , and then to *one* of the  $\mathbf{P}(\mathbf{A}^h)$ s, such that  $\mathbf{R}$  is in  $\mathbf{A}^h$  and, therefore, it can be identified with a *relation* with weight  $h$ . Observe that the union of the  $\mathbf{P}(\mathbf{A}^h)$ s is necessarily a disjunct one. If  $\mathbf{G}$  pertains to  $\mathbf{A}^h$  and  $\mathbf{A}^k$  then  $\mathbf{G}$  is a  $h$ -uple and, at the same time, a  $k$ -uple, what is impossible except if  $h=k$ .

By fixing the condition that  $\mathbf{R}$  is uniform, we obtain a function  $\mathbf{f}$  with weight  $h-1$ .

$\mathbf{Re}(\mathbf{E})$  e  $\mathbf{Fu}(\mathbf{E})$  are the *designed subsets* of  $\mathbf{A}^*$ , respectively called *the set of relations* and *the set of functions* of the structure  $\mathbf{E}(\mathbf{A})$ . The *constants* of  $\mathbf{E}(\mathbf{A})$  correspond to the case when the weight of the elements in  $\mathbf{Fu}(\mathbf{E})$  is  $\mathbf{0}$ . The *properties* in  $\mathbf{E}(\mathbf{A})$  correspond to the case when the weight of the elements in  $\mathbf{Re}(\mathbf{E})$  is  $\mathbf{1}$ . Also, the  $\mathbf{Re}(\mathbf{E})$ s with weight  $\mathbf{0}$  are the *sentences* of  $\mathbf{E}(\mathbf{A})$  (which are identified with *probability values*, or with truth value in the binary case).

$\mathbf{E}(\mathbf{A})$  is characterized by the sequence  $(\mathbf{A}, \mathbf{Re}, \mathbf{Fu})$ . The *syntax* of  $\mathbf{E}(\mathbf{A})$  is composed by symbols and rules (drawn from set theory) that allow its construction. The *semantics* of  $\mathbf{E}(\mathbf{A})$  is the set of rules that attribute *meaning* to elements of  $\mathbf{A}$ ,

to relations in  $\mathbf{Re}(\mathbf{E})$  and to functions in  $\mathbf{Fu}(\mathbf{E})$ , and also attribute a probability value to its sentences, when they occur.

We note that there is a large controversy about the "meaning of *meaning*" in the philosophy of language. In this paper we adopt the Tarskian theory, for which the meanings of terms and relations in one structure - the "object language"- are the corresponding elements and relations in an *isomorphic* structure - the "meta-language".

### 3 - Dynamical Processes, Determinism and Randomness

Consider a process generated by a mechanism  $\mathbf{d}$  that heats a mercury column  $\mathbf{m}$  in the interval  $(\mathbf{t0}, \mathbf{t1})$ . Among the properties that an observer can identify in this process is the temperature of  $\mathbf{m}$  in  $\mathbf{t0}$  and  $\mathbf{t1}$ , and other state variables of  $\mathbf{m}$ . The elements  $\mathbf{d}$  and  $\mathbf{m}$  can be considered as belonging to a set  $\mathbf{B}$ , where  $\mathbf{R}(\mathbf{d},\mathbf{m})$  is the relation of  $\mathbf{d}$  heating  $\mathbf{m}$ , and  $\mathbf{P}(\mathbf{m},\mathbf{t})$  is the temperature of  $\mathbf{m}$  in the instant  $\mathbf{t}$ . The structure  $\mathbf{E} = \mathbf{B}, \mathbf{R}, \mathbf{P}$  represents in a simplified way the state of the process of heating in a given instant (Lungarzo, 1970a, 1970b).

Along time, it is possible for  $\mathbf{B}$  to change, e.g., by adding a sample  $\mathbf{m}'$  of another material to the system heated by  $\mathbf{d}$ . In this case, the set  $\mathbf{B}$  is modified:  $\{\mathbf{d},\mathbf{m}\} \rightarrow \{\mathbf{d},\mathbf{m},\mathbf{m}'\}$ . Analogously, also  $\mathbf{R}$  is modified, since now  $\mathbf{d}$  heats both  $\mathbf{m}$  and  $\mathbf{m}'$ . Therefore, also the structure of the system - besides its states - undergo a temporal evolution. In this case, it is necessary to formalize this situation using several sets, representing different *universes*.

Given a sequence of sets  $\mathbf{B1}, \dots, \mathbf{Bn}$  composed of objects, we define a many-sorted structure  $\mathbf{E}$ , with  $\mathbf{Bi}$  universes, as:

$$\left\langle \mathbf{B}_1, \dots, \mathbf{B}_n; \left( \mathbf{R}_j^i \right)_{j=1, \dots, m(i)}^{i=1, \dots, n}; \left( \mathbf{F}_j^i \right)_{j=1, \dots, p(i)}^{i=1, \dots, n} \right\rangle$$

where:

1. The sets  $\mathbf{B}_i$  are finite;
2. For each  $i \in \{1, \dots, n\}$ ,  $\mathbf{R}_1^i, \dots, \mathbf{R}_{m(i)}^i$  are relations (or properties) in  $\mathbf{B}_i$ .
3. For each  $i \in \{1, \dots, n\}$ ,  $\mathbf{F}_1^i, \dots, \mathbf{F}_{p(i)}^i$  are functions in a proper Cartesian product of sets  $\mathbf{B}_{i_1}, \dots, \mathbf{B}_{i_q}$  in  $\mathbf{B}_i$ .

An object is a **dynamical structure** IFF:

1. is a family of many-sorted structures  $\{\mathbf{Et} \mid \mathbf{tT}\}$ , being  $\mathbf{T}$  a connected set of real numbers;
2. All the  $\mathbf{Et}$  are of the same kind, .
3. For each  $\mathbf{t}$ , the components (relations, functions) of  $\mathbf{Et}$  are functions (eventually, constant functions) of  $\mathbf{t}$ .

A *process* is a ordered pair formed by a dynamical structure and a closed finite interval of  $\mathbf{T}$ . If  $\mathbf{P}$  is a process, then  $\mathbf{a} < \mathbf{b}$ ,  $\mathbf{P} = , [\mathbf{a}, \mathbf{b}]$ . By  $\mathbf{P}(\mathbf{t})$  we indicate the dynamical structure  $\mathbf{Et}$ , for the instant  $\mathbf{t} [\mathbf{a}, \mathbf{b}]$ . As all the structures composing a process are of the same type , it can be called the *process type*. Such an uniformity does not block the possibility that a relation ceases to exist (in this case, it may be identified as a null relation , without changing .)

$\mathbf{P}(\mathbf{t})$  denotes the *state* of the process in the instant  $\mathbf{t}$ . The structure  $\mathbf{Et}$  represents the *configuration* of the system in the instant  $\mathbf{t}$ . Consider the process  $\mathbf{P} = (, [\mathbf{a}, \mathbf{b}])$ ; then,  $\mathbf{P}(\mathbf{a})$  is the *initial state* of  $\mathbf{P}$ , and  $\mathbf{P}(\mathbf{b})$  is the *final state* or *result* of  $\mathbf{P}$ .

A *knowledge landscape*  $\mathbf{C}$  is the structure  $\mathbf{P}, \mathbf{Ag}, \mathbf{S}$  where  $\mathbf{P}$  is a process,  $\mathbf{Ag}$  is a set of cognitive agents with potentially unlimited logical capabilities, and  $\mathbf{S}$  is a set of sentences that contain sufficient information for the description of  $\mathbf{P}$ . If  $\mathbf{P} = , [\mathbf{a}, \mathbf{b}]$ , then the process is *deterministic* IFF  $(\mathbf{Ag})$  (If knows the initial state of  $\mathbf{P}(\mathbf{a})$  with certainty, then the probability that knows the result  $\mathbf{P}(\mathbf{b})$  is  $\mathbf{1}$ ).

This is a purely epistemological definition of determinism. If the agents know the initial state with certainty, they are able to interpret all the sentences that describe this state and to deduce all the consequences from them; therefore they can calculate the results with maximum probability. As an example of deterministic process, let  $\mathbf{P}$  be the process of feeding a computer with the information necessary to factorize the number 2142 into primes, using Euclid algorithm and showing the result on the screen. In ideal conditions (without mechanical or electronic failures)  $\mathbf{P}$  is deterministic. Of course, the prediction of the final state depends on the information contained in  $\mathbf{S}$ . This information can change in another landscape.

A process is *not-deterministic* or *random* when there is at least one agent for whom the probability of knowing  $\mathbf{P}(\mathbf{b})$ , given  $\mathbf{P}(\mathbf{a})$  in  $\mathbf{C}$ , is lower than  $\mathbf{1}$ . Intuitively, it is possible to quantify randomness of a process, based on the quantity of agents failing to predict  $\mathbf{P}(\mathbf{b})$  with probability equal to  $\mathbf{1}$ , but this aspect is not relevant for our present argumentation.

Consider two *simultaneous* processes  $\mathbf{P}$  e  $\mathbf{Q}$ , such that  $\mathbf{P} = , [\mathbf{a}, \mathbf{b}]$  and  $\mathbf{Q} = '[\mathbf{a}, \mathbf{b}]$ , and the landscapes  $\mathbf{C} = \mathbf{P}, \mathbf{Ag}, \mathbf{S}$  e  $\mathbf{C}' = \mathbf{Q}, \mathbf{Ag}, \mathbf{S}$ . Now assume that:

1. (**Ag**) such that knows **P(a)** with certainty, but the probability that knows **Q(b)** é  $< 1$ , or
2. (**'Ag**) such that knows **Q(a)** with certainty, but the probability that knows **P(b)** é  $< 1$ .

In this case, the agents cannot predict with maximum probability the result of a possible interaction of **P** and **Q**. When this kind of situation occurs, we can say that **P** and **Q** are *independent* (while the *dependent* processes are those called "solidaires" by Cournot, 1838, §30)

Given two processes **P** and **Q**, such that  $P = \langle \cdot, [a, b] \rangle$  and  $Q = \langle \cdot, [c, d] \rangle$ , **P** is a *sub-process* of **Q**, (**P ? Q**) IFF:

1.  $c \leq a < b \leq d$ , and
2.  $\langle \cdot, [a, b] \rangle$  is a substructure of  $\langle \cdot, [c, d] \rangle$  for each  $t \in [a, b]$ .

When  $\langle \cdot, [c, d] \rangle = \langle \cdot, [c, d] \rangle$ , the sub-process is *normal*, and denoted by  $P \text{ ? }^* Q$ . If  $P \text{ ? }^* Q$  and

$c = a$ , then **P** is an *initial sub-process* of **Q**.

Now consider a finite sequence of deterministic processes, each pair of them being mutually independent, all of the same type  $\langle \cdot, [a, b] \rangle$ , but each one defined in a different landscape  $C_i: \langle P_1, \dots, P_h \rangle$ , such that  $i \in \{1, \dots, h\}$ ,  $P_i = \langle \cdot, I_i \rangle$ , being  $I_i$  non-disjunct time intervals.

If **P** is the *supreme* (upper bound) of the relation **?** of all **P<sub>i</sub>**, then a process **Q** is *generated* by **P<sub>i</sub>** processes IFF given  $t_0$  for some of the  $I_i$ , for all  $t \in t_0$  the structure of **Q** is an *effect* of the interaction of the structures **i** of **P<sub>i</sub>**.

#### 4 - Cournotian Processes

A *Cournotian* process **Q**, generated by the **P<sub>i</sub>**s, is a non-deterministic process at the union of the landscapes, such that  $Q \text{ ? } P$ . Intuitively speaking, a Cournotian process is a non-deterministic process generated by deterministic and independent processes, when the result of the interaction of the generating processes is not "larger" than the supreme of the sub-processes.

For instance, first consider a process **P** in landscape **C**. At  $t_0$  h of a chosen day, from the point  $x_0, y_0$  of the earth surface, a projectile **p** with mass **m** is shot with initial velocity  $v_0$ , forming an angle  $\theta$  with the earth's tangent. Also assume that it moves in a virtual vacuum. If **S** is the set of sentences that describe the

properties of the process, then any agent who knows with certainty the state  $P(0)$  can predict the state  $P(t)$  in  $t > 0$ , with probability 1. For  $t = 40$  seconds, the state  $P(40)$  is totally defined by the mass of  $p$  (the same) and the coordinates of  $p = (v_0 \cos 40), (v_0 \sin 40 - 800g)$ , being  $g$  the gravity force. Formally,  $P$  is the pair  $(\mathcal{S}, [0, 40])$  where  $\mathcal{S}$  is the structure containing  $p$ , the initial values and the empty space. The relation  $R(p, t, \dots)$ , defining the coordinates of  $p$  are time-dependent, and therefore the structure  $\mathcal{S}$  is a dynamical one.

Also a process  $P'$  occurs, in the landscape  $C'$ . At 0 h in the same day, a satellite  $s$  crosses the point  $x_1, y_1$  of the earth surface at the height  $h$ , with dynamical variables known for all  $Ag'$ . At the instant  $t$ , the agents  $Ag'$  can calculate the state  $s$  with probability 1.

Now consider that at the instant  $t'$  the projectile  $p$  collides with the satellite  $s$ . Agents from each landscape can calculate the instant  $t'$  when the collision occurs. However, each group does not know the variables of the other process. Therefore, the conjoint process  $Q$ , beginning at  $t'$  and leading to the separate drift  $p$  e  $s$  after the collision, cannot be predicted with probability 1, for both groups of agents. Therefore, in the conjoint landscape  $D$  the process  $Q$  is not deterministic.

Cournotian processes can be analyzed in the framework of Dynamical Systems Theory, including the informational properties of the systems, as proposed by Zadeh and Desoer (1963).

The model of a physical system  $M$  is a *dynamical system* IFF  $M = (I, O, f, h)$ , such that:

- 1)  $I, O$ , are sets and  $f, h$  are functions;
- 2) If  $s \in \Sigma$ , then  $s$  is a *state* of  $M$ ;
- 3) If  $f \in I$  and  $h \in O$ , then  $f$  and  $h$  are functions with domain in  $T$  (a non-null temporal interval);
- 4) The *range* of  $f$  is  $Ra(f)$ , i.e.  $f: T \rightarrow Ra(f)$  and  $h: T \rightarrow Ra(h)$ ;
- 5) The  $I$  functions are the *inputs* and  $O$  functions are the *outputs* of  $M$ ;
- 6) The  $f, h$  function is the *read-out*, which satisfies the properties:

$$6.a \quad \rho: (T \times \Sigma \times I) \longrightarrow O$$

$$6.b \quad \rho: (T \times \Sigma \times I) \longrightarrow O, \text{ such that}$$

$h(t) = df(\rho(t, s(t), f(t)))$  is the *response* of  $M$  in the instant  $t$ , when its state is

$s$  and the feeding input is  $f$ ;

We call  $\rho$  the *state transition function*, such that:

- 1)  $\rho: (\Delta \times \Sigma_0 \times I) \longrightarrow O$ , for  $\Delta = \{t, t', t_2 \mid t < t' < t_2\}$  and  $O$  being the set of all initial states;



2)  $t, t', s_0 \in \mathbf{0}, j \in \mathbf{I}$ , it follows that :  $t, t', s_0, j \longrightarrow s$ , such that  $s(t) = df (t, t', s_0, j)$  denotes the state produced by the input  $j$  at the instant  $t$ , from the state  $s_0$  that occurred at  $t'$ .

The state transition function must satisfy the following axioms:

- 1) *Transitivity*: for  $t, t' \in \mathbf{T}$ , such that  $t' < t, s_0 \in \mathbf{S}$ , and  $j, j' \in \mathbf{I}$ , if  $j(t) = j'(t)$  in  $[t, t']$ , then  $(t', t, s_0, j) = (t', t, s_0, j')$
- 2) *Associativity*:  $t < t' < t'' \in \mathbf{T}, s_0 \in \mathbf{S}, j \in \mathbf{I}$ ,  $(t'', t', (t', t, s_0, j), j) = (t'', t, s_0, j)$

The quantitative changes in a continuous dynamical system are usually presented by a system of linear or non-linear differential equations, referring to a function defined in  $\mathbf{R} \times \mathbf{D} \rightarrow \mathbf{D}$ , with  $\mathbf{R}$  being the set of real numbers and  $\mathbf{D}$  the set representative of the system to be modeled. The standard problem is to solve the equations, considering the initial and boundary conditions:

$$\begin{aligned} x \in \mathbf{D} \quad (0, x) &= x \\ t, t' \in \mathbf{S} \quad (t, (t', x)) &= (t+t', x) \end{aligned}$$

In the perspective of the axiomatic definition above, the transition function guarantees the existence of a solution for this problem. In the above example, the evolution of the projectile and the satellite can be represented by this kind of differential equations. They can be solved using the transition function ; its results can be read by the function and the set  $\mathbf{D}$  is  $\mathbf{R}^3$ . However, *the Cournotian process produced by the collision of the projectile with the satellite cannot be represented by a differential equation, because the solution is not a function.* If the process is studied as deterministic chaos, the deterministic phase leads to a bifurcation with an infinite branching of possible states. Therefore a Cournotian process, even in the case that the initial conditions are exhaustively known in the instant  $t = a$ , may become unpredictable in  $t = b$ .

In the study of Cournotian processes, the tools of Dynamical Systems Theory are able to exactly describe only the initial deterministic phase, before the independent causal lines meet. At this point, *a new phenomenon emerges*, one that cannot be deduced from the previous dynamics of the system.

After formalizing the Cournotian processes, it is possible to understand how to predict the result of this kind of process with a probability greater than chance. The agents, having only partial knowledge about what is going to happen after the causal lines meet, cannot deduce the outcome, but can make probabilistic inferences.

It is easy to show that this kind of computing is broader than the classical functional (or single deterministic Turing machine) approach. If the properties **P** are deduced from the theory **T(N)**, then, trivially, they can be inferred from the Cournotian process, since the deduction is the limit case when the probability value equals **1** (i.e., a sentence **e** is deducible from **e\*** if the probability of **e** is **1** always that the probability of **e\*** is **1**). Therefore, our approach can account for both the deductive and the non-deductive derivations.

## 5 – Structural Reasoning and Emergent Relations

The term *structuralism* is used in several areas of scientific methodology and epistemology, referring to formal procedures used with problems that require the consideration of *non-trivial* structures. In this sense, *structuralism* has been used to refer to theories in Economy, Anthropology, Linguistics and Psychology. In this paper, we conceive structures in a pragmatic framework, as *theoretical abstractions or models* used in the context of empirical science, to organize the available data and make predictions.

There are at least three different concepts of *structuralism*:

- 1) in Chomsky's classical studies of grammar, efforts were directed to find *combinatorial structures* that operate on linguistic elements, following formal rules, in order to explain the categories of natural language. These efforts concentrated on the syntactic aspects of language, leaving semantics on a secondary plane;
- 2) in the context of philosophy of mind, structuralism is related to "bottom-up" approaches, aiming to explain psychological functions from neuroscientific theories and data, in opposition to the "top-down" methodology of cognitive science that is focused on psychological functions (see Fodor, 1983);
- 3) in the context of the sciences of life, structuralism refers to biological structures (macromolecules, cells, tissues, organs, body systems), a case when it is *not* opposed to the consideration of functions (i.e., *biological functions*), since biological structures and functions are complementary concepts (structures refer to the components of living systems, and functions refer to the activities that such structures display in time).

The Chomskyan concept of structuralism (item 1, above) is based on a previous distinction between syntactic and semantic aspects of natural language, made in the context of Model Theory. The structures used as models also have their own syntax; however, their *application* to theories is not necessarily syntactic. There are other aspects of semantic evaluation; for instance, when a statement **e** is satisfied in a model **M**, this means that **e** is *true* in **M** (in the Tarskian approach, "truth" is an *isomorphism* between structures).

Eventually, the inferences used in Model Theory may have been considered as syntactic by some authors, since for *first order* logic the syntactic and semantic approaches lead to the same results, according to Henkin's Completeness Theorem. However, from a conceptual perspective both are different.

The complementarity of structural and functional aspects can be achieved by considering higher-order systems, as assumed in non-reductive explanatory strategies. In first-order systems, functions are *internal* to one structure (in our previous formulation, one **Fu(E)** for each **E**). However, for higher-order systems it is possible to construct relations that link different and/or hierarchically ascending structures. For instance, if **E** and **E'** are first-order structures, it is possible to construct a second-order relation **r: E E'**.

Consider **M** to be a set of cognitive properties, and **N** a neuroscientific theory that is true. **S** is a model of the brain and its interactions with the body and environment. Also assume that the standard model of **N** is isomorphic to **S**, and that there is a set of statements **M** expressing the properties of mental phenomena that cannot be predicted/deduced from **N**. These are *strongly emergent* properties, according to our previous definition.

The reason why mental phenomena are not deducible from **S** can be made explicit straightforwardly. Cognitive phenomena are expressed by a class of predicates that is semantically separated from the predicates used to express biophysical processes; there is no middle term, and therefore these classes of predicates – as usually stated – belong to separate semantical categories. Therefore, there is not a valid syntactic pathway to deduce statements about mental properties from statements about biophysical properties, unless:

- a) *bridge principles* are formulated, providing a connection between (at least) one biophysical predicate and one mental predicate, as proposed by Reductionism (Bickle, 2003), or
- b) mental predicates are reformulated using biophysical predicates, as proposed by Eliminative Materialism.



It is of central importance for our proposal to consider a semantic-structural, non-syntactic form of reasoning that is the basis for ascription of probability values: the *judgment of similarity* (see Gärdenfors, 2000). The notion of *similarity* expresses the existence of *partial correspondences* between structures (see Pereira Jr., 1999) and therefore operates in the domain of a second-order relation  $\mathbf{r}$ . Prediction about  $\mathbf{E}'$  based on knowledge of  $\mathbf{E}$  is called *reasoning with similarities*.

In the study of the brain/mind system, we consider  $\mathbf{E}$  as the brain structure (with respective internal functions, described by neuroscientific theories) and  $\mathbf{E}'$  as the mental structure (with respective internal functions, described by psychological theories). As we adopt a non-reductive explanatory strategy, we do not identify both structures (as in Identity Theories) and we do not try to syntactically deduce  $\mathbf{E}'$  from  $\mathbf{E}$  (even with the usual inclusion of bridge principles).

We propose the second-order relation  $\mathbf{r}$  to be conceived as a semantic relation based on similarity. At this moment we are not able to formalize this model and will limit our exposition to five sketchy examples where the formalization could be applied.

## 6 - Some Examples from Neuroscience

Emergent cognitive functions, not deducible from a (hypothetically complete) neuroscientific theory, can be predicted by structural similarities between brain activities and mental experiences. We give five examples to illustrate how reasoning by similarity works in the context of cognitive neurobiology. The examples display similarities between alterations in brain activity and the corresponding alterations in mental activity. Such similarities advance one step beyond the merely temporal correlations found by current techniques in cognitive neuroscience, helping to explain the relative success of biological psychiatry in the treatment of several kinds of psychopathology.

First, animals with a deficit in protein CaMKII (Calmodulin-dependent Protein Kinase II) activity display a deficit in memory formation (Wang et al., 2003). Based on this experimental finding, it is possible to infer that the lack of a function  $\mathbf{f}$  in  $\mathbf{S}$  implies the lack of a function  $\mathbf{f}'$  in  $\mathbf{M}$ . This reasoning is *induction by vicariance*.

A second example is: a decrease in serotonin levels predicts the onset of depression. Once serotonin is a neuromodulator that increases the efficacy of

synapses, a decrease in serotonin levels would also decrease the efficacy of synaptic communication. Although we do not know exactly what synaptic communication has to do with *mood*, a mental phenomenon, we can find a similarity between serotonin decrease and a decrease in mental disposition. This reasoning is *induction by similarity*.

The third example is an experiment that produced an *increase* in molecular function. It is well known that the membrane receptor NMDA is involved in the capacity of associative learning. Genetically modified mice with over-expression of the NMDA receptor are predicted to display improved learning capabilities, since this receptor works as a coincidence detector, providing neuronal excitation upon receiving two excitatory pulses in a narrow time window. Associative learning is a mental function that consists basically of connecting different stimuli. It is not the case that the pulses received by the NMDA receptor at each neuron correspond to the stimuli to be associated; however, its physiological function has some degree of similarity with the mental function. This is also a case of *induction by similarity*.

The fourth example is a case when *increasing* the quantity of one kind of molecular component leads to a *decrease* of mental activity. Also in this case there is *induction by similarity*, but this time with inverse proportionality. It is well known that the transmitter GABA and its receptors have a physiological function of inhibiting neuronal activity. Substances that perform the same function of GABA (binding to the benzodiazepinic receptors) are used in Psychiatry as tranquillizers, in the treatment of anxiety and psychoses. Anxiety is a mental phenomenon, having properties that cannot be deduced from biophysical processes in the brain. However, we can find a similarity between the physiological function of promoting neuronal inhibition and the mental function of tranquillizing.

The last example is the approximation of the dynamics of a signaling ion ( $Ca^{++}$ ) with mental processes as learning, memory formation and consciousness (see Koch, 2003, and also Robertson, 2002, for a similar proposal regarding  $Ca^{++}$  waves in astrocytes). Koch's reasoning is based on a *timing analogy*. The timing of  $Ca^{++}$  entry in synaptic ion channels and binding with receptors in the post-synaptic density is analog to the timing of such mental processes, so it was inferred to be related with them.

In the five examples, the structural correlations do not follow logical or biophysical laws, and could be taken as pieces of scientific fiction, except for the fact that all of them have been experimentally demonstrated. The second and fourth examples correspond to widely used pharmacological drugs. The first



example was extensively reviewed in Bickle (2003). The third example was confirmed by the breeding “Smart Mice” (see Tang et al., 1999).

## 7 - Concluding Remarks

As in the classical concept of *chance* advanced by Cournot (1838), here we considered emergence processes as resulting from the meeting of independent causal lines (i.e. semi-deterministic processes) generating a product that cannot be described as a *function* (but as a mere *relation*) of the multiplicity of factors involved in the process. Therefore, biological and cognitive functions would correspond to mathematical *relations*.

The occurrence of emergence in Cournotian processes is predictable - although not deductible - by semantic reasoning based on similarities. If biological and mental processes derive from Cournotian processes, we need knowledge about structural properties of living bodies and brains to perform such reasoning and predict emergent mental properties. When researchers attempt to construct life or mentality artificially, the possibility of making these inferences can be lost.

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