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**Abstract** The uncertainty characteristics of reverse logistics make the revenue's distribution of reverse logistics is more complex, this text forms a concise game analysis between driving side(government) and executing side(enterprises) among reverse logistics, mainly discussing the game problem of revenue distribution from the supply chain only consisting of one collector and one processor among reverse logistics by the way of game analysis. And we get some conclusions as following: revenue proportion is reducing when produce processing cost of processor is increasing; supply chain of independent decision is low efficiency, when other conditions remain unchanged, the value of processor's processing unit cost is greater, the efficiency is lower.

**Key words** Reverse logistics; Game analysis; Supply chain; Revenue

## 1 Introduction

Since the 1990s, the world economy and material civilization are developing rapidly, but due to the enormous economic growth mode with the way of extensive growth, most countries still develop at the old path, firstly developing then administrating and the direct consequence is that the environment pollution, especially the global Climate still has a trend of increase in these years, according to experts' prediction that the CO<sub>2</sub> levels reached 450PPM /M<sup>3</sup>, the repairing function of the air by itself is lost (namely the state is irreversible), the earth will be into the ice age in advance, now the concentration of CO<sub>2</sub> from the air has reached 375PPM/M<sup>3</sup>, almost close the critical value of 450PPM/M<sup>3</sup>, thus, environmental problem is very pressing,. Along with the enhancing of environmental protection consciousness, sustainable development and the reverse logistics is attracting more attention.

Reverse logistics, hasn't formed a unified definition at home and abroad yet, the concept described earliest in 1981, Lambert and Stock think that reverse logistics is different from normal channels of shipment of the products of reverse flow. In the 1990s U.S. logistics management association (CLM) (1992) published two important articles on reverse logistics, the paper gives the definition of reverse logistics, it is a kind of logistics activity containing product return, material substitution, the disposal and recycle, processing, waste disposal, maintenance and remanufacture process<sup>[1]</sup>. Stock pointed out the importance of reverse logistics affecting the business and Society<sup>[2]</sup>. Kipick puts forward the principles and practice of reverse logistics, discussed the reuse and recycling opportunity<sup>[3]</sup>.

The research on the reverse logistics began in the mid 1990s. In China it begins in recent years. According to the domestic and foreign literatures published on reverse logistics, current researches get some achievements, such as literatures related this main theme following: they study the game on both sides of enterprise, government and consumers or among them. The former is taken for the research by Yu Liying, the later is the game analyzed by Fu Pengwei and Qin Yanhua, et al.

Yu Liying(2004) discusses the equilibrium and the outcome of the game, and puts forward the countermeasures on building the reverse logistics system from the government and the enterprise<sup>[4]</sup>. Using the game theory, Li Jinyong(2007) builds up a two-person game model between government and enterprise, analyzes the behavior of government and enterprise<sup>[5]</sup>, Li Jiping, Chen Shuqing and Li Junping(2009) give the calculating formula of the enterprise's subsidy rate from government after analyzing the relationship on both government and enterprise during the process of constructing the logistical system<sup>[6]</sup>.

While Fu Pengwei, Li Jun, and Wang Jirong(2006) analyze the interactional relations among enterprises ,government and consumers using game theory, and finally give the government some advice on instructing the resource utilization behaviors of enterprises and consumers using tax policies, which are intended to promote the implement of reverse logistics<sup>[7]</sup>. QinYanhua(2006) considers whether enterprises implement the reverse logistics is a multi-game between enterprises, customers and government<sup>[8]</sup>, Shu Siliang(2008) analyzes the government and the enterprise about the implementation of recovery reverse logistics, and proposes the most superior strategy of the government guiding on the implementation of enterprise recovery reverse logistics<sup>[9]</sup>. Furthermore, some researchers study on

related subjects' game among the reverse logistics, such as: Li Ying(2005) thinks that the environmental protection tax play a key role in implement of reverse logistics<sup>[10]</sup>.

Seen from above review, there are few of literatures studying on revenue distribution of related subjects on reverse logistic at present, however, how to do revenue distribution in the formation of reverse logistics becomes a important problem.

This paper briefly does the game analysis between reverse logistics' driving side (government) and executing side (enterprises), and then analyzes the game problem of revenue distribution from the supply chain only consisting of one collector and processor among reverse logistics.

## 2 Model Construction and Analysis

### 2.1 The game analysis between driving side (government) and executing side (enterprises)

Hypothesis: the cost of treatment from enterprise (gambling side 1) for  $A_1$  if not discarding, the cost of treatment for society is  $A_2$  if discarding, for the enterprise does not need to undertake the social costs, enterprise's net utility of discarding waste for  $A_1$ .

The government (gambling side 2) establishes rules and regulations to punish some one who discards waste at  $A_3$ . As shown in Figure 1:

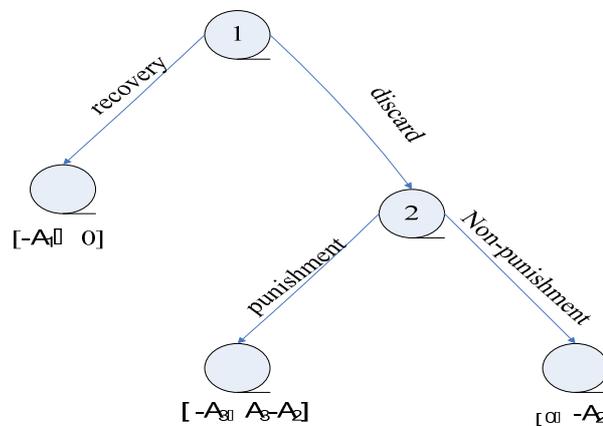


Figure 1 The Analysis of Game Between Government and Enterprises

If the government in the form of regulations outgoes a believable threat: "discarding old products will be punished," and  $A_1 > A_3$ , it can bring a good supervision function to enterprise implementation of reverse logistics.

### 2.2 The game analysis among related subject of reverse logistics

According to the formation mode of reverse logistics, it is divided into two types: return reverse logistics and reverse logistics of recycling. Return reverse logistics, namely the downstream customers will not conform to order products returned to upstream suppliers, its process flow and conventional products is the contrary. According to the source of goods returned, it is divided into manufacturing return, commercial return, manufacturing product recall, warranty return; according to use recycled way into reuse, return reverse logistics will be divided as remanufacturing and recycling, destroyed, etc.

The forming process of reverse logistics can be expressed as below:

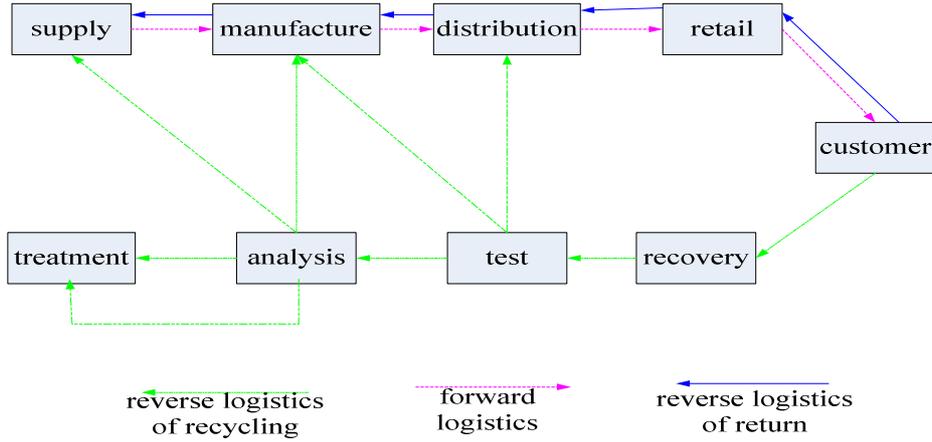


Figure 2 Flow Chart of Reverse Logistics

Here we will first simplify reverse logistics chain into one collector and one processor, and then analyze revenue distribution problem on both sides.

### 2.2.1 Model description

Collector will be responsible for collecting EOL (end of life) products and transferred to processor, processor is responsible for products processing and marketing materials, then gains.

According to the relationship between collecting product quantity and collecting price wrote by MukhoPadhyay<sup>[11]</sup>, then considering random factor, assuming the collector with product supply  $q$ ,  $q = a + bp + \mathcal{E}$ ,  $p$  is collecting products' price,  $a, b > 0$ ,  $\mathcal{E}$  is uniform distribution of random variables,  $\mathcal{E} \in [0, B]$ ,  $B > 0$ , density function  $f(x) = 1/B$ , the mathematical expectation of random variable  $\mathcal{E}$  of  $E\mathcal{E} = B/2$ . the unit collecting cost from reverse logistics is  $c_1$ ,  $c_1$  includes the cost of collection, storage and transportation, but  $c_1$  does not include collecting price  $p$ . Processor's processing unit cost is  $c_2$ .  $c_2$  consists of processing and sales, and depreciation costs. Waste products after processing gets the unit profit is  $r$ ,  $r$  including product sales parts, materials may come from renewable income and government subsidies. Generally speaking, processor's processing ability is limited, we assume that the biggest processing ability to handle things is  $Q$ ,  $Q > 0$ .

Collector and processor through the proportion of revenue sharing contract, the contract parameters for sharing proportion and product purchase prices. Processor's decisions will share the revenue; The proportion of revenue for collector is  $\lambda$ ,  $0 \leq \lambda \leq 1$ , The proportion of revenue for itself is  $1 - \lambda$ . According to the proportion of shares collector decides the price of collecting products ( $p$ ). This collector and processor's decisions based on profit maximization respectively as processor is the leaders, the collector is the contractor's followers. Obviously, due to the limited ability of processor, collector cannot make higher collecting price (though the price higher, the product will be collected more). In order to guarantee the collector and processor's profit is nonnegative, we assuming  $p + c_1 \leq \lambda r$ ,  $c_2 \leq (1 - \lambda)r$ . For convenience, assuming  $1 \leq B$ ,  $1 \leq Q$ . Assuming collector and processor are risk-neutral, the following is calculated at the expected profits. Let's first consider collector and processor's cooperation of decision-making, and then analyze Stackelberg game situation.

### 2.2.2 Cooperative decision-making

When the collector and processor of reverse logistics do their centralized decision-making, the expected profits of the whole system is  $\pi_c(p)$ :

$$\begin{aligned} \pi_c(p) &= -(p + c_1 + c_2)E[\min\{q, Q\}] + rE[\min\{q, Q\}] \\ &= (r - p - c_1 - c_2)E[\min\{q, Q\}] \end{aligned} \quad (1)$$

(2)

Introducing variables  $z$ ,  $z = Q - (a + bp)$  (2), Put (2) into (1) gets  $\pi_c(p) = (r - p - c_1 - c_2)(Q - \frac{z^2}{2B})$

$$\text{Change } p \text{ to } z, \text{ then } \pi_c(p) \text{ becomes } \pi_c(z) = (r - \frac{Q - a - z}{b} - c_1 - c_2)(Q - \frac{z^2}{2B}) \quad (3)$$

Ask first-order derivative  $\pi_c(z)$ ,  $\frac{d\pi_c(z)}{dz} = \frac{2BQ - 2z(br - bc_1 - bc_2 - Q + a) - 3z^2}{2bB}$

$$\text{Let } \frac{d\pi_c(z)}{dz} = 0, \text{ then } 3z^2 + 2z(br - bc_1 - bc_2 - Q + a) - 2BQ = 0 \quad (4)$$

The solution of equation (4) is

$$z_c = \frac{-(br - bc_1 - bc_2 - Q + a) \pm \sqrt{(br - bc_1 - bc_2 - Q + a)^2 + 6BQ}}{3} \quad (5)$$

Because  $z = Q - (a + bp) = Q - (q - \varepsilon)$ ,  $Q \geq q$  and  $\varepsilon \in [0, B]$ , from this we know that  $z \geq 0$ , so

$$(5) \text{ We can only take } z_c^* = \frac{-(br - bc_1 - bc_2 - Q + a) + \sqrt{(br - bc_1 - bc_2 - Q + a)^2 + 6BQ}}{3} \quad (6)$$

$$\frac{d\pi_c^2(z)}{dz^2} = \frac{-(br - bc_1 - bc_2 - Q + a + 6z)}{2bB} < 0 \quad \text{that, } \pi_c(z) \text{ is } Z\text{'s concave function. Put } (6)$$

into  $z = Q - (a + bp)$ , We get:

Proposition 1: The optimal price of centralized decision-making making by collector and processor is:

$$p_c^* = \frac{(br - bc_1 - bc_2 + 2Q - 2a) - \sqrt{(br - bc_1 - bc_2 - Q + a)^2 + 6BQ}}{3b} \quad (7)$$

### 2.2.3 Independent decision: Stackelberg game

The decision of collector:

For a given proportion of income distribution, for the expected profits function of collector, we use  $\pi_E(p)$  for presentation.

$$\pi_E(p) = -(p + c_1)E[\min\{q, Q\}] + \lambda r E[\min\{q, Q\}] = (\lambda r - p - c_1)(Q - \frac{z^2}{2B}) \quad (8)$$

Proposition 2: On any given  $\lambda$ , the response function of collector is:

$$p_E^*(\lambda) = \frac{2Q - 2a + b\lambda r - bc_1 + \sqrt{(b\lambda r - bc_1 - Q + a)^2 + 6BQ}}{3b} \quad (9)$$

$$\text{Put(2)into(8),then } \pi_E(p) \text{ into } \pi_E(z), \pi_E(z) = (\lambda r - \frac{Q - a - z}{b} - c_1)(Q - \frac{z^2}{2B}) \quad (10)$$

$$\text{Ask first-order derivative, Let } \frac{d\pi_E(z)}{dz} = 0, \text{ then } 3z^2 + 2z(b\lambda r - bc_1 - Q + a) - 2BQ = 0 \quad (11)$$

Because  $z \geq 0$ , so the solution of equation (11) is

$$z_E^*(\lambda) = \frac{-(b\lambda r - bc_1 - Q + a) + \sqrt{(b\lambda r - bc_1 - Q + a)^2 + 6BQ}}{3} \quad (12)$$

$$\frac{d\pi_E^2(z)}{dz^2} = \frac{-(b\lambda r - bc_1 - Q + a + 3z)}{bB} = -\frac{b(\lambda r - c_1) - bp + 2z}{bB} \quad \text{According to the original}$$

hypothesis,  $p + c_1 \leq \lambda r$ ,  $z \geq 0$ , so  $\frac{d\pi_E^2(z)}{dz^2} < 0$ , Namely the expected profits function of collector is a

concave function. By type (12) and (2), we get the response function for collector is:

$$p_E^*(\lambda) = \frac{2Q - 2a + b\lambda r - bc_1 + \sqrt{(b\lambda r - bc_1 - Q + a)^2 + 6BQ}}{3b} \quad (9) \quad \text{We can find the collecting price } p_E^*$$

and the profits of unit product  $r$ , the gains of the collector  $r$ , The proportion of the profits  $\lambda$  have some relationship with their costs.

Proposition 3: Established in other factors remain unchanged,  $p_E^*$  increasing along with  $r$  increases and  $\lambda$  increases, but reducing along with  $c_1$  increases. (Proof elliptically)

Comparison with the collector price of cooperative decision-making  $p_c^*$  (type (7)) and independent decision-making  $p_E^*$  (type (9)), we get: Proposition 4, when  $\lambda = 1 - c_2 / r$ ,  $p_c^* = p_E^*$ .

Proposition 4 means, for independent decision-making, when processor distribute all their revenue that come from deducted their cost, total revenue of the reverse logistics chain equal with total revenue of cooperative decision-making.

The decision of processor:

When collector entity set collection product price for  $p_E^*$ , the corresponding collecting amount is determined by  $q = a + bp + \varepsilon$ , processor set the distribution proportion  $\lambda$  to make their profit maximization. Use  $\pi_D(\lambda)$  represents processor's expected profit function.

$$\begin{aligned} \pi_D(\lambda) &= -c_2 E[\min\{q, Q\}] + (1-\lambda)rE[\min\{q, Q\}] = [(1-\lambda)r - c_2](Q - \frac{z_E^*}{2B}) \\ &= \frac{[(1-\lambda)r - c_2][2BQ + z_E^*(b\lambda r + a - Q - bc_1)]}{3B} \lambda \end{aligned} \quad (13)$$

Proposition 5, the distribution proportion of processor when they achieve profit maximization  $\lambda^*$ ,  $\lambda^*$  decided by the next type:

$$[(1-\lambda)r - c_2][(b\lambda r + a - Q - bc_1) \frac{\partial z_E^*}{\partial \lambda} br] - r[2BQ + z_E^*(b\lambda r + a - Q - bc_1)] = 0 \quad (14)$$

$$\text{where } z_E^*(\lambda) = \frac{-(b\lambda r - bc_1 - Q + a) + \sqrt{(b\lambda r - bc_1 - Q + a)^2 + 6BQ}}{3} \quad (12)$$

Comprehensive Proposition (2) and (5), we can get proposition (6): Collector and processor's decisions exists Stackekberg equilibrium ( $p_E^*$ ,  $\lambda^*$ ),

$$\text{Especially, } p_E^* = \frac{2Q - 2a + b\lambda r - bc_1 + \sqrt{(b\lambda r - bc_1 - Q + a)^2 + 6BQ}}{3b} \quad (9), \text{ but } \lambda^* \text{ is given by (14).}$$

### 3 Conclusion

On the basis of the above analysis, the following conclusions can be drawn:

Firstly, Type (14) Show the revenue proportion  $\lambda^*$  that processor distributing to collector relates to its produce processing cost  $c_2$ , and  $\lambda^*$  is reducing along with  $c_2$  increasing, namely  $\frac{\partial \lambda^*}{\partial c_2} < 0$ .

Secondly, When the revenue proportion of collector  $\lambda^* < 1 - \frac{c_2}{r}$ , then  $p_E^* < p_c^*$ .

Finally, Independent decision supply chain is low efficiency, when other conditions remain unchanged, the value of  $c_2$  is greater, the efficiency is lower.

### References

- [1] De Brito M P, Dekker R. A Framework for Reverse Logistics[R]. ERIM Report Series Research in Management, ERS-2003-045-LIS, 2003
- [2] Stock J R, Oak Brook IL. Reverse Logistics Council[J]. Logistics Management, 1992: 1-3
- [3] Kopichi R. Reuse and Recycling-Reserve Logistics Opportunities[J]. Oak Brook IL. Council of Logistics Management, 1993: 141-146
- [4] Yu Liying. The Game Analysis in Reverse Logistics[C]. Proceedings of the 7th China Academic Exchanges on Operational Research, 2004 (1): 229-234 (In Chinese)
- [5] Li Jinyong, Liu Wei, Cheng Guoping. Game Analysis about Reverse Logistics Between Government and Enterprise[J]. Journal of the Northwest University of the Agriculture and Forestry (Social Science Edition), 2007, (03): 54-57 (In Chinese)
- [6] Li Jiping, Chen Shuqing, Li Junping et al. Research on Reverse Logistics' Building Based on Game Theory[J]. Market Modernization, 2009, (07): 151 (In Chinese)
- [7] Fu Pengwei, Li Jun, Wang Jirong et al. Multi-player Game Analysis of Reverse Logistics Construction[J]. Chinese Scientific and Technological Papers Online, 2006, (03): 232-238 (In Chinese)

- [8] Qin Yan-hua, Cao Xi-yu. Game Analysis of the Enterprise Reverse Logistics[J]. Ecological Economy, 2006, (04): 77-79 (In Chinese)
- [9] Shu Siliang, Liu Xiangli. Game Theory Application in the Government's Guiding on the Implementation of Enterprise Recovery Reverse Logistics[J]. China Market, 2008, (23): 45-47 (In Chinese)
- [10] Li Yin. Game Model Among Reverse Logistics[J]. Science Technology and Industry, 2005, (11): 62-63 (In Chinese)
- [11] Smar K. Mukhopdhyay, Robert SetoPutrol. Optimal Return Policy and Modular Design for Build-to-order Products[J]. Journal of Operations Management, 2005, 23: 496-506