Research on the Multilevel Indicator Evaluation Model of Online Learning Based on Fuzzy Set

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Abstract Learning evaluation is an effective way to ensure the quality of online education. However, most of online learning evaluation models are not fully consider the characteristics of online learning. This paper, from the view of the characteristics of online learning, establishes an index system of evaluation of the online learning’s effect. Besides, weight of every factor affecting online learning is calculated by using entropy method. At the same time, a model based on entropy weight of the fuzzy comprehensive evaluation is established and its calculation is given out. At last, the learning effects of a learner are evaluated and its result is analyzed by this model.

Key words Entropy; Online learning; Weight; Fuzzy Comprehensive evaluation

1 Introduction Learning for a long life is the current development trend of social education. Construction of modern network education is the primary means of lifelong learning system. Internet offers an ideal learning environment for distance education. With the launch of online learning, its’ quality has aroused extensive concern. Quality of online education is the key to development for online learning, and also the core competence for the long-term development of online education. Learning evaluation is effective way to ensure the quality of online education. Learning evaluation is also the core of online education evaluation system.

Currently, the evaluation of online learning is concerned by domestic and foreign scholars. For example, Ellen.B researched the development trend of learning evaluation [1]. Jonghee Huh had researched the evaluation about reading activities in e-learning [2]. Moreover, the evaluation of online learning was used in online education by some countries. Meanwhile, the development of online learning evaluation system also been researched. These studies mainly focus on the design and implementation of online learning evaluation system. Some universities have developed these systems. Such as Web-CT, WISH, Virtual-U, Black-Board, Course-Info, Path-Ware [3]. However, most of these studies used the traditional education and learning evaluation system to evaluate the online learning. In traditional education and learning evaluation system, written or online tests are the main forms of evaluation, learning evaluation was made by teachers, learners’ level were often measured by learners’ test results. It can not fully consider the characteristics of online learning. It was difficult to adapt to rapid development of online learning. Therefore, it needs to establish new indicators evaluation system to adapt to the feature of online learning.

2 Evaluation Indicator System of Online Learning Learning behavior was a multi-dimensional and multi-level learning style, which was carried out by means of the Internet; it was a behavior of student self-discipline and self-control. Actors had sufficient autonomy to determine learning goals, learning schedule, learning strategies, learning resources, as well as the occurrence and changes of the learning behavior. We could describe the learning behavior by the following model.
Figure 1  Behavior Model of Online Learning

Through this model, we could summarize the main internal factors that have impact on online learning such as preparation for learning, attitude of learning, learning ability of learners and information exchange capabilities. In addition to internal factors, the contents of online course and online test also affected the evaluation of online learning behavior. We summarized these factors as external factors which were impact of online learning. Therefore, evaluation indicator system of online learning could be divided into 6 first grade indicators and 27 second grade indicators.

3 Online Learning Assessment Model Based on Entropy Weight Fuzzy Comprehensive Evaluation Method

3.1 The model of entropy calculation

In the online learning assessment, different indicators will have different impacts on the effect of online learning. Therefore, we need to assign weight to every indicator. In the traditional method, we directly assign every indicator’s weight by experience. This method is very simple, but is difficult to be objective and reasonable. Meanwhile, it is difficult to ensure consistency in the process of critical thinking. In this paper, concerning the characteristics of the online learning, we invite several learner’s partners and teachers as experts to mark, so as to make the weights to be in line with the actual situation. However, different evaluators have different understanding to the factors which impact on the effect of online learning, because of different background, therefore, weight has some divergence. In order to get the weight of reflecting objective requirements on the subjective evaluation basis, we use method of entropy assessment to analyze and process the result objectively. In other words, it is to calculate the weight of every indicator relative to the weight of upper indicators through the inner relations among those indicators. In this method, subjective judgment is combined with objective calculation. And the calculation is as following [5].

There are \( m \) evaluators and \( n \) indicators, \( y_{ij} \) is the score from evaluator \( i \) evaluate indicator \( j \). Then:

\[
Y = \begin{bmatrix}
  y_{11} & y_{12} & \cdots & y_{1n} \\
  y_{21} & y_{22} & \cdots & y_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  y_{m1} & y_{m2} & \cdots & y_{mn}
\end{bmatrix},
\]

\( y_j^* \) is the highest score of indicator \( j \). It is the value which reflects the effect of assessing every indicator. \( d_{ij} \) is the approaching degree between \( y_{ij} \) and \( y_j^* \).


\[ d_{ij} = y_{ij} / y^*_j \]  \hspace{1cm} (1)

According to the definition of entropy, \( H \) is the entropy of \( m \) evaluators and \( n \) indexes.

\[ H = - \sum_{j=1}^{m} \sum_{i=1}^{n} d_{ij} \ln d_{ij} \]  \hspace{1cm} (2)

<table>
<thead>
<tr>
<th>Table 1  Evaluation Indicator System of Online Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparation for learning ( U_1 )</td>
</tr>
<tr>
<td>Basic knowledge of discipline ( U_{11} )</td>
</tr>
<tr>
<td>The convenience of learning time ( U_{12} )</td>
</tr>
<tr>
<td>Plan of learning ( U_{13} )</td>
</tr>
<tr>
<td>Computer ability of learners ( U_{14} )</td>
</tr>
<tr>
<td>The economic situation of learners ( U_{15} )</td>
</tr>
<tr>
<td>Attitude of learning ( U_2 )</td>
</tr>
<tr>
<td>Participate in online teaching and learning activities actively ( U_{21} )</td>
</tr>
<tr>
<td>Study on Internet by self ( U_{22} )</td>
</tr>
<tr>
<td>Without cheating in the online test ( U_{23} )</td>
</tr>
<tr>
<td>Submitting the work timely ( U_{24} )</td>
</tr>
<tr>
<td>Ability of Learning ( U_3 )</td>
</tr>
<tr>
<td>Using Internet search tools ( U_{31} )</td>
</tr>
<tr>
<td>Using online database resources ( U_{32} )</td>
</tr>
<tr>
<td>Using forum resources ( U_{33} )</td>
</tr>
<tr>
<td>Using multimedia resources ( U_{34} )</td>
</tr>
<tr>
<td>Answer questions on the Web ( U_{35} )</td>
</tr>
<tr>
<td>Information exchange capabilities ( U_4 )</td>
</tr>
<tr>
<td>Published the views of relevant courses in the forum ( U_{41} )</td>
</tr>
<tr>
<td>Discuss experiences with other partners in the forum ( U_{42} )</td>
</tr>
<tr>
<td>Giving the suggestion on the online teaching ( U_{43} )</td>
</tr>
<tr>
<td>completing the relevant issues with other learning partners ( U_{44} )</td>
</tr>
<tr>
<td>Contents of online course ( U_5 )</td>
</tr>
<tr>
<td>The correlation of knowledge ( U_{51} )</td>
</tr>
<tr>
<td>The continuity of knowledge modules ( U_{52} )</td>
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<tr>
<td>The scalability of knowledge ( U_{53} )</td>
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<tr>
<td>The integrity of knowledge ( U_{54} )</td>
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<tr>
<td>The stability of knowledge structure ( U_{55} )</td>
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<tr>
<td>Contents of online test ( U_6 )</td>
</tr>
<tr>
<td>The difficulty of online tests ( U_{61} )</td>
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<tr>
<td>The knowledge coverage of online test ( U_{62} )</td>
</tr>
<tr>
<td>Amount of test questions ( U_{63} )</td>
</tr>
<tr>
<td>The knowledge extension of the online test ( U_{64} )</td>
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</tbody>
</table>

The uncertainty of the index is decided by the conditional entropy \( H' \)

\[ H' = - \sum_{j=1}^{m} \left( \frac{d_{ij}}{d_j} \right) \ln \left( \frac{d_{ij}}{d_j} \right), d_j = \sum_{i=1}^{n} d_{ij} \] \hspace{1cm} (3)

\[ H'_{\text{max}} = \ln m, \text{ conditional entropy is normalized by } H'_{\text{max}}, e(d_j) \text{ is the entropy which reflects the importance of indicator.} \]
\[ e(d_j) = H'/\ln m \]  
\[ W_j \] is the weight of index.  
\[ w_j = [1 - e(d_j)] / \sum_{j=1}^{n} [1 - e(d_j)] \]

### 3.2 Constructing model based on entropy weight fuzzy comprehensive evaluation

#### 3.2.1 Constructing a set of evaluation factors

According to the properties of the evaluation factors, a set \( U \) can be established. In this paper, \( U \) contains 6 first grade indicators as: \( U = \{U_1, U_2, \ldots, U_6\} \). Every first grade indicator contains several second grade indicators as: \( U_j = \{U_{j1}, U_{j2}, \ldots, U_{jk}\}, (j = 1, 2, \ldots, 6) \)

Reviews of each index are divided into \( m \) levels. In this paper, 5 grade reviews are used to evaluate every indicator of the model. \( V = \{V_1, V_2, V_3, V_4, V_5\} \). \( V \) is a set of reviews. In the set \( V_1 \) is the worst, the score of \( V_1 \) is between 0 and 20. \( V_2 \) is worse, its score is between 20 and 40. \( V_3 \) is normal, its score is between 40 and 60. \( V_4 \) is better, its score is between 60 and 80. \( V_5 \) is the best, its score is between 80 and 100. Those ‘best’, ‘better’, ‘normal’, ‘worse’, ‘worst’ represent the effect of the online learning \([6]\).

#### 3.2.2 Determining the weight of every indicator

The weight of the index of first level is calculated by the model of entropy calculation. \( W \) is a set of weight as \( \{W_1, W_2, W_3, W_4, W_5, W_6\} \). \( W_j \) is a set of sub-indicator, \( W_j = \{W_{j1}, W_{j2}, \ldots, W_{jk}\} \).

Process of calculation is in Section 3.1.

#### 3.2.3 Constructing evaluation matrix

\( R_j \) is the fuzz matrix from \( U_j \) to \( V \).

\[ R_j = \begin{bmatrix} r_{i11} & r_{i12} & \cdots & r_{i15} \\ r_{i21} & r_{i22} & \cdots & r_{i25} \\ \cdots & \cdots & \cdots & \cdots \\ r_{ik1} & r_{ik2} & \cdots & r_{ik5} \end{bmatrix} \]

In this paper, \( r_{ijp} \) is the membership grade of \( U_j \) to \( V_p \), \( V_p \) is element of \( V \). According to the statistics of the experts’ scores, we can get \( U_j \), there are \( m_{j1} \) reviews belonged to \( V_1 \), \( m_{j2} \) reviews belonged to \( V_2 \), \( m_{j3} \) reviews belonged to \( V_3 \), \( m_{j4} \) reviews belonged to \( V_4 \), \( m_{j5} \) reviews belonged to \( V_5 \), at last, we can obtain the value of \( r_{ijp} \).

\[ r_{ijp} = m_{ijp} / \sum_{p=1}^{5} m_{ijp} \]  

#### 3.2.4 Calculating the value of model

Calculating \( R_i \) with fuzzy method \( C_i \) is obtained. \( C_i \) is the vector, which reflects the membership grade of \( U_i \) to \( V \).

\[ C_i = W_i \star R_i. \]  
\[ C_i = (C_{i1}, C_{i2}, C_{i3}, C_{i4}, C_{i5}) , C_{ip} = \min\{1, \sum_{j=1}^{5} W_j r_{ijp} \} (p=1, 2, \ldots, 5) \]
Judgment matrix $R$ is composed with all the vectors $C_i$.

$$R = \begin{bmatrix}
C_1 \\
C_2 \\
\vdots \\
C_6
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & \ldots & C_{15} \\
C_{21} & C_{22} & \ldots & C_{25} \\
\vdots & \vdots & \ddots & \vdots \\
C_{61} & C_{62} & \ldots & C_{65}
\end{bmatrix}$$ (8)

$C = W \cdot R = (W_1, W_2, \ldots, W_6) \cdot (C_1, C_2, \ldots, C_6)^T = (c_1, c_2, \ldots, c_6)$ (9)

$$c_p = \min \{1, \sum_{i=1}^6 W_i C_{ip}\} \ (p = 1, 2, \ldots, 5) \quad \text{when} \quad \sum_{p=1}^5 c_p \neq 1,$$

normalized $C,$

$$\text{s.t.} \quad \sum_{p=1}^5 c_p = 1, \quad c_p = c_p / \sum_{p=1}^5 c_p.$$

At last, $u = 90 \times c_1 + 70 \times c_2 + 50 \times c_3 + 30 \times c_4 + 10 \times c_5$, $u$ is the result of model, the bigger the value of $u$ is, the better the effect of the online learning is.

### 4 Case Studies

Now there is an online learner, 6 experts evaluate the effect of online learning. They score all the indicators from different levels and the indicators are presented in Table 1. The range of score is 1 to 10. $Y$ is the score of the relative importance of first-level indicators $Y_1, Y_2, Y_3, Y_4, Y_5, Y_6$. $Y_1, Y_2, Y_3, Y_4, Y_5, Y_6$ is the score of the relative importance of second-level indicators which belong every first-level indicators.

$$Y = \begin{bmatrix} 5 & 9 & 8 & 5 & 6 & 8 \\ 5 & 8 & 6 & 7 & 7 & 6 \\ 5 & 6 & 5 & 6 & 7 & 5 \\ 4 & 7 & 7 & 5 & 6 & 6 \\ 5 & 6 & 6 & 5 & 7 & 6 \\ 4 & 9 & 9 & 9 & 9 & 7 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 5 & 8 & 5 & 5 & 6 \\ 7 & 6 & 4 & 6 & 5 \\ 6 & 7 & 5 & 6 & 7 \\ 7 & 7 & 5 & 5 & 5 \\ 8 & 6 & 6 & 5 & 6 \\ 5 & 5 & 5 & 4 & 6 \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} 6 & 8 & 7 & 6 \\ 7 & 6 & 8 & 6 \\ 6 & 7 & 7 & 5 \\ 7 & 9 & 9 & 5 \\ 8 & 6 & 8 & 7 \\ 6 & 5 & 7 & 8 \end{bmatrix}$$

According to Formula (1), (3), (5), we can calculate the relative weights of first-level indicators $W$ is the weight which is from $U_i$ to $U$. $W = \{W_1, W_2, \ldots, W_6\} = \{0.086, 0.232, 0.228, 0.190, 0.083, 0.179\}.$

The same as $W$:

$$W_1 = \{0.307, 0.219, 0.155, 0.180, 0.138\}, \quad W_2 = \{0.130, 0.407, 0.151, 0.312\},$$

$$W_3 = \{0.219, 0.195, 0.247, 0.201, 0.137\}, \quad W_4 = \{0.292, 0.292, 0.255, 0.161\},$$

$$W_5 = \{0.187, 0.263, 0.217, 0.145, 0.188\}, \quad W_6 = \{0.251, 0.291, 0.297, 0.161\}.$$

The experts score all the 27 indicators referred in Table 1. The range of score is 0 to 100. The score higher the risk higher. From the view of statistics, the result is reasonable. According to Formula (6), the judgment matrixes are obtained.
According to Formula (7), $C_1$ can be calculated

$C_1 = W \cdot R_1 = \{0.0495, 0.1402, 0.2316, 0.2901, 0.2876\}$

The same as $C_1$, $C_2, C_3, C_4, C_5, C_6$ also can be calculated. According to Formula (8), Judgment matrix R can be calculated.

$R = \begin{bmatrix}
C_1 & 0.0495 & 0.1402 & 0.2316 & 0.2901 & 0.2876 \\
C_2 & 0.1407 & 0.1688 & 0.2537 & 0.2442 & 0.1962 \\
C_3 & 0.1218 & 0.1751 & 0.2354 & 0.2778 & 0.1889 \\
C_4 & 0.0708 & 0.1708 & 0.2416 & 0.2839 & 0.2329 \\
C_5 & 0.1 & 0.1217 & 0.2263 & 0.2855 & 0.2477 \\
C_6 & 0.1 & 0.1709 & 0.2548 & 0.2703 & 0.2040
\end{bmatrix}$

According to Formula (9),

$C = W \cdot R = \{0.1043, 0.1643, 0.2427, 0.2710, 0.2147\}$

The value of the effect of A's online learning maybe 10.43% between 0-20, 16.43% between 20-40, 24.27% between 40-60, 27.10% between 60-80, 21.47% between 80-100.

At last, $u = 10 \times c_1 + 30 \times c_2 + 50 \times c_3 + 70 \times c_4 + 90 \times c_5$

$= 0.1043 \times 10 + 0.1643 \times 30 + 0.2427 \times 50 + 0.2710 \times 70 + 0.2147 \times 90 = 56.391$

The value of the effect is 44.036. Therefore, the effect of A's online learning is normal.

5 Conclusions

Evaluation of the online learning is a complicated process. The effect of online learning is influenced of all kinds of factors. While the weight of these factors is determined, we can make a reasonable assessment for the effect of online learning so that learning of students' and teaching methods of teachers' could be adjusted in time. At the same time, an indicator system of online learning assessment is founded. Weight of every factor is calculated by entropy theory. Through the establishment of fuzzy comprehensive evaluation model to evaluate the effect of the online learning, teachers can understand the current status of online learner better, and then teachers can manage the online learning more effectively.

References

